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Money in a Theory of Banking

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Abstract

We explore the connection between money, banks, and aggregate credit. We start with a simple "real" model without money, where banks make loans repayable in goods and depositors hold claims on the bank payable on demand in goods. Aggregate production may be delayed in the economy. If so, we show that the level of ongoing bank lending, and hence of aggregate future output, can decrease with increases in the real repayment due on deposits: ceteris paribus, the higher the amount due, the more likely there will be insufficient goods, given the delay, to pay depositors, and the more new lending has to be curtailed to make up the shortfall. Thus a temporary delay in production can be exacerbated by banks into a more permanent reduction of total output. A number of inefficiencies including bank failures can result if deposits turn out to be too high. We then introduce money in this model. We show that if demand deposits are repayable in money rather than in goods, banks can be hedged against production delays: under certain circumstances, the price level will rise with delays in production, reducing the real value of the deposits banks have to pay out. But demand deposits payable in money can expose the banks to new risks: the value of money can fluctuate for reasons other than delays in aggregate production. Because deposits are convertible into money on demand, a temporary rise in money demand immediately boosts the interest rate banks have to pay depositors, which in turn boosts the real amounts banks have to repay them. This increase in the real deposit burden can again lead to the curtailment of bank lending and even bank failures. The way to combat these contractionary effects is to infuse more money into the banking system. Our analysis thus makes transparent how changes in the supply of money can work through banks to affect real economic activity, without invoking sticky prices, reserve requirements, or deposit insurance. It also suggests how bank failures could lead to a fall in prices and a contagion of bank failures, as described by Friedman and Schwartz (1963).

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What is the connection between money, banks, and aggregate credit? When can expansionary monetary policy lead to expanded bank credit? And when can expansionary monetary policy help avert bank failures? These are the questions that motivate this paper.

We start with a "real" model where all contracts are denominated in goods. A bank is an intermediary, which has special skills that enable it to lend to firms that are hard to collect from. The bank finances these loans by issuing demandable claims. In Diamond and Rajan (2001), we show why such an arrangement allows banks to fund potentially long term projects while allowing investors to consume when needed.

Unfortunately, demandable claims expose the banking system to the risk that there might be a mismatch between the production of consumption goods and the real amount promised to the depositors. Even if the mismatch is merely due to delay in production and not because of any impairment of the long-term production possibilities of the economy, a temporary aggregate shortage of consumption goods, also termed a real "liquidity shortage", can be amplified by the banking system. Banks will cut short long dated projects (that is, curtail credit). Banks may fail and failures may propagate contagiously through the system even if depositors have the most optimistic beliefs possible (unlike, say, in models like Diamond and Dybvig (1983)).

All this is shown in an economy where all contracts, including demand deposits, pay in goods, and there is no money. The repayment on deposits is assumed fixed over the next instant and cannot be an explicit function of realizations of individual bank or aggregate conditions. Simply allowing the repayment on bank deposits to vary with aggregate prices like the real interest rate will not completely resolve the problem. But there is another potential fix: In practice, bank deposits and loans typically repay money, not goods.[1] What would happen if we introduced money and bonds in this model, and allowed for deposit contracts denominated and repayable in money (i.e., *nominal deposits*)? Would the systemic problems we have documented

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[1] Though deposits are denominated in foreign currencies in some countries, which essentially makes them real from the perspective of that country's citizens.

vanish? Can monetary policy play any useful role? It is to these important questions that we turn in this paper.

We focus on two essential sources of value for money. First, money is a claim on the government and may have intrinsic value from the ability of citizens to pay taxes with it. Second, it facilitates transactions: some goods may be sold only for cash as in the cash-in-advance literature (see Clower [1967] and Lucas-Stokey [1987]). These could be goods that are illegal like drugs, goods such as services that are sold in transactions where the seller may seek to keep his identity hidden from tax authorities, or just goods encountered serendipitously where the relative cost of establishing a credit transaction may be too high. Depending on circumstances, the "fiscal" value of money may exceed or be dominated by its "transactions" value.

Suppose now that banks issue nominal deposits whose repayment is denominated in money. To the extent that the value of money in terms of goods falls when the current aggregate supply of goods is low, deposits denominated in money offer banks a natural hedge against aggregate fluctuations. And there is reason why this might be the case – the present value of taxes on production falls if production is delayed so the fiscal value of money falls. Similarly, it is plausible that transactions fall when aggregate production is delayed so the transactions value of money could also fall. The real value paid out on nominal deposits will then be state contingent in a way that reduces adverse effects on bank loan portfolios and bank health.

However, perfect state contingency is, as we show, a fairly special idealization. In fact, it is possible that the value of money be unrelated or even negatively correlated with current aggregate production: for instance, the transactions value of money may be determined by opportunities outside the formal domestic economy, or may even be negatively correlated with aggregate production.[2] When banks issue nominal demandable deposits, they are left particularly

[2] If illegal goods and informal services require cash-in-advance, there is no reason they should be positively correlated with aggregate formal production. In fact, if more workers lose formal employment and enter the informal sector, there could be a case for arguing for negative correlation between aggregate production

exposed to fluctuations in the purchasing power of money: Since depositors can withdraw money on demand, in a period when cash transactions are very lucrative (for example, because the supply of money is low relative to available cash goods) banks will be forced to push up the interest rates offered on demandable deposits significantly so as to keep all depositors from withdrawing. In turn, this will increase the real repayment obligations of the banks, potentially without limit. Because banks are special and their loans are illiquid, their higher repayment obligations will affect real activity. If monetary conditions thus do not fluctuate in lock-step with real activity, nominal deposit contracts can exacerbate the risks banks face.

Our analysis then suggests a channel through which monetary policy can affect credit and thus aggregate economic activity. By increasing the money supply available for transactions when the transactions demand is high (and by committing to provide monetary support in the future when needed), the monetary authority keeps the price level stable, thus limiting depositor incentives to withdraw, and consequently limiting both nominal interest rates and future real repayment obligations of banks (this point is related to Champ, Smith and Williamson (1996), though we detail a number of differences later). Banks then will respond by continuing, rather than curtailing, credit to long-term projects.

Our view of the monetary transmission mechanism could then be termed a version of the *bank lending channel view* (see Bernanke and Gertler (1995) or Kashyap and Stein (1997)) for comprehensive surveys) but with an important difference. According to the traditional lending channel view, monetary policy affects bank loan supply, which in turn has an independent and significant effect on aggregate economic activity (this does not preclude any independent effects of movements in the discount rate or changes in the quality of corporate balance sheets). Three assumptions have been thought to be key to the centrality of banks in the transmission process: (i) binding reserve requirements tie the issuance of bank demand deposits to the availability of

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and cash transactions. Similarly, if a temporary downturn prompts asset sales, a negative correlation could again emerge.

reserves (ii) banks cannot substitute between demand deposits and other forms of finance easily so they have to cut down on lending when the central bank curtails reserves (iii) client firms cannot substitute between bank loans and other forms of finance, so they have to cut down on economic activity.

The concern with the traditional view of the bank lending channel is that as reserve requirements have been eliminated for almost all bank liabilities except demand deposits, the argument that banks will find it difficult or expensive to raise alternative forms of financing to demand deposits becomes less persuasive (see, for example, the critique by Romer and Romer (1990), though see Stein (1998) who argues that demand deposits are still special because they are insured). But there does seem to be strong evidence that monetary policy has effects on bank loan supply (Kashyap, Stein, and Wilcox (1995), Ludvigson (1996)), has greater effect on banks at times when their balance sheets look worse (Gibson (1996) and has the greatest effect on the policies of the smallest and least credit worthy banks (Kashyap and Stein (2000)).

In contrast to traditional models of the lending channel, our model does not rely on reserve requirements or on deposit insurance, or even on sticky prices. An increase in the money supply increases financial liquidity, which alleviates the real liquidity demands on banks, which then allows them to fund more long-term projects to fruition. These effects will be most pronounced for constrained banks in bad times. Thus it is perhaps best to term ours the *liquidity* version of the lending channel of transmission.

The rest of the paper is as follows. In section I, we describe the framework, in section II we describe the problems with real deposit contracts and the circumstances under which nominal contracts can improve upon them. In section III, we introduce money and examine how aggregate activity and bank credit is affected by shortages of real and financial liquidity. In section IV we examine how monetary policy is transmitted in our model and then we conclude.

I. The Framework

1.1. Agents, Preferences, Endowments, Technology.

We will first lay out the "real" model then overlay it with a simple role for money. Consider an economy with three types of risk neutral agents: investors, entrepreneurs, and bankers and three dates: 0, 2, and 4 (the intermediate dates are introduced later). Investors get utility only from near-term consumption, that is, their utility is the sum of consumption at, or before, date 2. All other agents get equal utility from long-term consumption also, so their utility is the sum of consumptions at all dates including date 4.

Investors are each endowed initially with a fraction of a unit of good. No other agent is endowed with goods. Goods can be stored at a gross real return of 1 or they can also be invested in projects.

Each entrepreneur has a project, which requires the investment of a unit of good before date 0. It pays off C produced goods at date 2 if the project produces *early* or C at date 4 if the project is delayed and produces *late*. There is a shortage of endowments of goods initially relative to projects that can be invested in.

1.2. Projects and the non-transferability of skills

The primary friction in the model is that those with specific skills cannot commit to using their human capital on behalf of others. This implies that they will not be able to borrow the full value of the surplus they can produce with an asset or sell the asset for the amount they can produce with it. Both projects and loans to projects will thus be illiquid because of the inalienability of human capital (see, Hart and Moore (1994)).

Since entrepreneurs have no endowments, they need to borrow to invest. Each entrepreneur has access to a banker who has, or can acquire during the course of lending, knowledge about an alternative, but less effective, way to run the project. The banker's specific knowledge allows him to (make the credible threats that will enable him to) collect γC from an entrepreneur whose project just matures.[3] No one else has the knowledge to collect from the entrepreneur.

Regardless of whether a project is early or late, the banker can also *restructure* the project at any time to yield c in date-2 goods – intuitively, restructuring implies stopping half finished projects and salvaging all possible produced goods from them. Restructured projects can be collected by anyone. We assume

$$c < 1 < \gamma C < C , \qquad (1.1)$$

Since no one other than the bank has the specific skills to collect from the entrepreneur, the loan to the entrepreneur is illiquid in that the banker will get less than γC if he has to sell the loan before the project matures. Any buyer will realize that the banker will extract a future rent for collecting the loan, and the buyer will reduce the price he pays for the loan accordingly. In fact, bank loans are so dependent on the banker's specific skills for collection (that is, they are so illiquid) that the banker prefers restructuring projects to selling them.[4]

Since there is a shortage of endowment relative to projects, only a select few entrepreneurs get a loan from their respective banks to buy a unit of the good from investors. Entrepreneurs will have to promise to repay the maximum possible on demand, γC, to obtain the loan.

1.3. Financing Banks

We analyze the general equilibrium effects in an economy where banks borrow goods to lend to entrepreneurs by issuing a mix of demand deposits and bank capital. Our positive results do not depend on the reasons these forms of finance are used. In our previous work (Diamond and Rajan (2000, 2001)), we argued that the demandable nature of deposit contracts introduces a collective action problem for depositors that makes them run to demand repayment whenever they anticipate the banker cannot, or will not, pay the promised amount. Thus deposits, unlike

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[3] See Diamond and Rajan (2001) or Hart and Moore (1994) for an extensive form game with this outcome.
[4] See Diamond and Rajan (2003) for the easily satisfied conditions that lead to this outcome.

other claims, cannot be renegotiated down in an orderly fashion. Because bankers will lose all rents when their bank is run, they will not attempt to renegotiate deposits down. Deposit financing introduces a rigidity in the bank's required repayments, which enables the banker to commit to repay if he can (that is, avoid strategic defaults by passing through whatever he collects to depositors), but it exposes the bank to destructive runs if he truly cannot pay (it makes non-strategic default more costly): when depositors demand repayment before projects have matured and the bank does not have the means of payment, it will be forced to restructure projects to get c immediately instead of allowing them to mature and generate γC.

Such rigidity can be tempered by raising part of the funds through claims that can be renegotiated. In particular, we focus on the issue of capital where the renegotiation process leads the banker and capital holders to split the residual surplus after deposits have been paid (see the extensive form game in Diamond and Rajan (2000)). Assuming equal division, capital will be paid $\frac{v-d}{2}$ where v is the present value of the bank's assets in their best use from capital's perspective and d is the level of deposit repayments. So the virtue of capital is that its value adjusts to the underlying value of the bank's assets, its cost is that the bank cannot raise as much up front against its future receipts since the banker absorbs a surplus that increases in the level of capital. We assume that capital has to be fraction k of the present value of bank assets (this could be because of un-modeled uncertainty the bank faces on its loans or an explicit capital requirement imposed by regulators) so that the bank can raise only

$$\frac{v}{1+k} \tag{1.2}$$

against its assets by issuing a mix of deposits and capital.[5] We add bank capital to the model only to understand its effects, the key results are unchanged if we leave it out.

1.4. Uncertainty

Each bank faces an identical pool of entrepreneurs before date 0. But at date 0, the fraction of the funded projects that turn out to be early could differ. A bank's projects could all turn out to be early (type G bank) or only a fraction $\alpha^B < 1$ could be early (type B bank). The fraction of banks of type G in state s is $\theta^{G,s}$ and the fraction of early projects for the B type bank is $\alpha^{B,s}$. In what follows, we will suppress the dependence on the state for notational convenience. All quantities will henceforth be normalized by the total initial endowment of goods.

1.5. Timing.

Before date 0. Investors are endowed with goods. They invest them in competitive banks in return for bank claims (deposits and capital) that make them better off in expectation than storage. Each bank offers to repay d_0 on demand per unit of good they get from investors and a commensurate value on capital. Banks lend the goods to entrepreneurs in return for a promise to repay γC on demand. Entrepreneurs invest the goods in projects.

Date 0. Uncertainty is resolved: everyone learns which entrepreneurs' projects are early and which are late, and thus what fraction α^i of a bank *i*'s projects are early. We assume that depositors in a given bank run at date 0 only if they anticipate it cannot survive at date 2 given its distribution of borrower types and given the market clearing interest rate that will prevail at date 2. In other words, we do not consider panics where depositors run at date 0 only because they

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[5] From the definition, we have $k = \frac{\frac{1}{2}(v-d)}{\frac{1}{2}(v+d)}$ where the numerator on the right hand side is the value of capital and the denominator is the value of capital plus maturing deposits. Therefore, the total amount that can be pledged to investors out of the amount the bank collects is the denominator, which on substituting for d works out to $\frac{v}{1+k}$.

think other depositors will run, regardless of date-2 fundamentals.[6] We allow collective action problems but not coordination failures.

A run will force the bank to first pay out all the goods it has, and then restructure projects (first late projects then early ones) to generate the goods needed to pay depositors. If no run occurs, the bank decides how to deal with each late project – whether to restructure it if proceeds are needed before date 4, or perhaps get greater long run value by rescheduling the loan payment from date 2 to date 4 and keeping the project as a going concern.

Date 2. Entrepreneurs with early projects will produce C, and repay the bank γC. This leaves them with $(1-\gamma)C$ to invest as they will. The bank obtains repayments from early entrepreneurs, proceeds from restructured late projects, and raises additional funds through a mix of deposits and capital from early entrepreneurs and other bankers with surplus. It must meet capital requirements for these new issues. Investors present their claims and are paid out of the funds (goods) the bank raises, which they consume.

Date 4. Late entrepreneurs repay banks and banks repay date-2 investors (early entrepreneurs and other bankers). Entrepreneurs and bankers consume.

II. Aggregate Liquidity Shortages and Bank Credit.

We showed in Diamond and Rajan (2001) that banks and their fragile liability structures are essential to facilitate the flow of credit from investors with uncertain consumption needs to entrepreneurs who have hard-to-pledge cash flows. If investors lent directly, acquired collection skills, but wanted to consume at an interim date, they would have to sell their loans at a huge discount. Far better to hold demand deposits on a bank and let the bank acquire the collection skills. If the investor wants to consume at an interim date, the bank will pay him and refinance by borrowing from others (early entrepreneurs) who have a surplus. In this way, the bank does not

[6] In other words, while we assume that each depositor expects the others in the same bank to choose a withdrawal that is an individual best response to others' actions (so we assume non-cooperative actions where individual incentives may not lead each depositor to maximize the welfare of the whole), they all agree to choose the set of Nash actions that make them best off.

interrupt the late, but valuable, project while also allowing the investor to consume a larger amount when he desires consumption.

Unfortunately, the liability structure of banks leaves them exposed to temporary aggregate shortages of goods. A small delay in the supply of goods relative to their demand can propagate through bank credit contraction and bank failures into a longer term, and more widespread adverse shock to production. This is what we show now. Once we see how the "real" model works, we will be able to see clearly the role of money in this model.

2.1. Banks' maximization problem

Because the G type banker's projects all mature at date 2, he will have enough to repay investors provided $\gamma C \geq d_0$ (and under no circumstances will the initial level of deposits be set higher than γC, else banks would not be able to repay under any circumstances and would all be run).[7] Let the gross real interest rate between date i and date j be r_{ij}. Because everyone is indifferent between consumption at date 0 and at date 2 and no real investments between those dates offer a higher return, r_{02} is 1.

Now consider the B type banker's maximization problem if the bank is expected to survive after uncertainty is revealed at date 0. He has the following decision problem: What fraction of late projects does he restructure at date 0 so as to maximize his consumption while constrained by the necessity to pay off all bank claimants? This problem can be written as

$$\max_{\mu^B} \quad \alpha^B \gamma C + \mu^B (1-\alpha^B) c_1 + (1-\mu^B)(1-\alpha^B) \frac{\gamma C}{r_{24}} \qquad (2.1)$$

[7] It is easily shown (see Diamond and Rajan (2003)) that the bank will not store any goods if the expected return on lending to projects dominates the return on storage. Thus if lending is profitable, there will be no storage.

s.t. $$v^B(\mu^B, r_{24}) \geq \frac{\max\limits_{\mu'} v^B(\mu', r_{24}) + d_0}{2} \quad (2.2)$$

To see why this is his maximization problem, start with the constraint (2.2). Let the B type banker restructure μ^B of his late projects (since all of a G type banker's projects are early, $\mu^G=0$). The value he can raise in date-2 consumption goods is

$$v^i(\mu^i, r_{24}) = \alpha^i \gamma C + \mu^i(1-\alpha^i)c + (1-\mu^i)(1-\alpha^i)\frac{\gamma C}{(1+k)r_{24}} \quad (2.3)$$

The first term is the amount repaid by the α early entrepreneurs whose projects mature at date 2. The second term is the amount obtained by restructuring late projects. The third term is the amount the bank can raise (in new deposits and capital – see (1.2)) against late projects that are allowed to continue without interruption till date 4, where r_{24} is the real interest rate banks offer on deposits between dates 2 and 4 (this need not be 1 because the initial investors prefer date 2 consumption over date 4 consumption).

In bargaining over how much they have to be paid, capital holders have the right to pick a level of restructuring that maximizes what they get.[8] This can be different from the level chosen by the banker. Hence the repayment to capital and deposits is $\frac{\max\limits_{\mu'} v^i(\mu', r_{24}) + d_0}{2}$.[9] Therefore, (2.2), the constraint on the banker, is simply that the resources he can raise should exceed the required repayment.

The banker's objective is to maximize the present value of his total consumption. Since the bank's repayment to initial investors is invariant to the actual amount of restructuring chosen,

[8] Intuitively, if capital has the right to make a take-it-or-leave-it offer to the banker, it will demand all the value the banker generates net of deposits when the assets are put to their best use *from the perspective of capital*.

[9] Capital gets (v-d)/2 while depositors get d.

the banker is the residual claimant of repayments by entrepreneurs on bank loans and his objective can be changed to maximizing the present value of the total repayments on his loans, as in (2.1). Note that this exceeds (2.3), the amount he can raise at date 2, because the banker cannot raise money against his own prospective future rents. The solution to the banker's problem is

Lemma 1: Let $R = \frac{\gamma C}{(1+k)c}$ and $\overline{R} = \frac{\gamma C}{c}$. If $r_{24} < R$, the banker will not restructure any projects (so that $\mu^B=0$) and the bank will survive provided it can pay date-0 depositors – provided $v^B(0, r_{24}) \geq d_0$. If $R < r_{24} < \overline{R}$, the banker will restructure the minimum fraction μ^B such that $v^B(\mu^B, r_{24}) \geq \frac{v^B(1, r_{24}) + d_0}{2}$. Finally, if $r_{24} \geq \overline{R}$, the banker will restructure all late projects and the bank will survive provided $v^B(1, r_{24}) \geq d_0$.

The lemma indicates the fraction the banker restructures increases with the real interest rate. At low interest rates ($r_{24} < R$), the banker gets more value and can raise more by *continuing* late projects. At high interest ($r_{24} \geq \overline{R}$) the banker gets more value and can raise more by *restructuring* late projects. But at intermediate rates, the banker has an incentive to continue, though he can pay claimants more by restructuring. This is because the banker gets rents from continued late projects that bank claimants do not see (the last term in (2.1) is greater than the last term in (2.3)). So he will restructure the minimum that will be necessary to pay off claimants.

Other decisions are less complicated. The entrepreneur's production decision is entirely passive – he produces in due course if his project is not restructured by the bank beforehand. If he produces, he repays the bank. Early entrepreneurs invest their residual goods (of $(1-\gamma)C$) in the bank at date 2 if it can credibly promise to repay $r_{24} \geq 1$.

2.2. Equilibrium Condition and aggregate credit.

The only price at date 2 in this "real" model is the relative price of date 2 to date 4 consumption: the real interest rate, r_{24}. Since investors can express their purchasing power only with their claims on the bank, the demand for consumption (real liquidity) is the total real value of their claims on the bank. The real interest rate ensures the total investor demand for goods at date 2 is weakly less than the supply of goods.

$$\theta^G \frac{\max_{\mu'} v^G(\mu', r_{24}) + d_0}{2} + (1-\theta^G)\frac{\max_{\mu'} v^B(\mu', r_{24}) + d_0}{2} \leq \left[\theta^G C + (1-\theta^G)\left(\alpha^B C + (1-\alpha^B)\mu^B c\right)\right]$$

(2.4)

The real side of our model should now be fairly clear. The adverse shocks in our model are merely delays in the timing of production – adverse shocks to the quantities produced would only exacerbate the problems. Even though the total production possibilities of the economy over dates 2 and 4 do not change with increases in the fraction of B banks, ($1-\theta$), and the fraction of their late projects, ($1-\alpha^B$), the amount of consumption goods available at date 2 (aggregate real liquidity) falls. Given an excess of demand over supply for liquidity, the real interest rate will rise to clear the market. Supply rises as banks restructure more late projects. The fraction of late projects a B type bank continues, $1-\mu^B$, could be thought of as a measure of the credit it extends, so credit falls. Demand falls as a higher real interest rate reduces the real value of the B type bank's capital, and thus reduces the purchasing power claimants have. Hence an incipient liquidity shortage is alleviated by an increase in the real rate, which increases the supply (and reduces credit) while reducing the demand for liquidity.

Let the total supply of consumption goods not be enough to meet the total demand without some restructuring by B type banks. If both types of banks survive and the B type banks restructure a positive fraction of their late projects, we have the following comparative statics

Proposition 1:

For a given level of deposits issued at date –1,

(i) Equilibrium credit extended at date 2, $(1-\mu^B)$, decreases with an decrease in the fraction of G type banks, θ^G.

(ii) Equilibrium credit extended at date 2, $(1-\mu^B)$, decreases with a decrease in the fraction of projects of B type banks that are early, α^B.

(iii) For every θ^G, there is an α^* (possibly 0) such that B type banks are insolvent iff $\alpha^B < \alpha^*$.

(iv) For a given θ^G and α^B, the equilibrium credit extended at date 2 falls with the outstanding level of real deposits, d_0.

Proof: See appendix.

The proposition indicates that a decrease in the intrinsic supply of consumption goods at date 2 (a decrease in either θ^G and α^B) or an increase in demand (an increase in d_0) leads to a curtailment in credit. Essentially, in a situation of aggregate liquidity shortage, banks are squeezed between a rock (non-negotiable deposits) and a hard place (hard to sell loans). They survive only by “liquefying” projects. Ironically, this arises because in the normal course demand deposits commit the bank to collect (and thus enable it to borrow against) the illiquid loans.

2.3. Bank failures.

If severe enough, aggregate delays can also cause banks to fail. Recall that if a bank is expected to fail, depositors run and demand payment immediately on the revelation of uncertainty at date 0. Since projects pay at date 2 at the earliest, and since the bank obtains more from restructuring rather than selling the illiquid project loans, all projects including the early ones are restructured, and the proceeds paid to depositors. Failure is inefficient because early projects could have produced C in a timely manner to satisfy the consumption needs of date-0 investors, but now produce only c. The collective action problem inherent in demand deposits is now destructive for it forces the costly production of consumption goods when none is really needed!

When all contracts are real, there is only one price – the real interest rate – which can adjust to clear markets. The system may have insufficient degrees of freedom to adjust to an adverse shock, resulting in the stark consequences we have documented (as we will argue in the example later, even deposit contracts contingent on the real interest rate will not be sufficient to eliminate these problems, what we really need is deposit contracts with full state contingency). This real model is not without practical interest – when a country's banking system has deposits denominated in foreign exchange (or if the country were on the Gold standard), it is as if the banks have issued real deposits. But our primary purpose in investigating the real model is to pinpoint the links between the "real" economy and money. As we will see, one link emerges when banks issue *nominal* deposits, that is, deposits denominated in cash. Money affects the price level, which in turn will affect the real payout on nominal deposits. As we have just seen the required real payout on deposits can affect the level of real activity. This is the link we now make clear.

III. Money and Banking

We focus on two natural sources of value for money. First, money can serve as a store of value; we introduce this into our finite horizon economy by assuming that money (and any maturing government liability) can be used to pay future taxes. This is one anchor for the value of money, which we shall call the *fiscal* demand. Second, money facilitates certain transactions that by their very nature are unexpected, opportunistic, small-volume, or worth concealing so that the use of formal credit is ruled out. This is the *transactions* demand for money. Both demands will be important in understanding the link between money and banking.

3.1. Transactions Demand

Start first with the *transactions* demand. We introduce one more agent, the dealer, who obtains equal utility from consuming a unit at any date. The dealer receives an endowment of a perishable good, which can be sold only for cash (to fix ideas, the good is his labor, and he does not report this income to the tax authorities so he accepts only cash). “Early” dealers obtain an endowment q_1 of this *cash* good at date 1 while “late” dealers obtain q_3 at date 3. One unit of this cash good produces the same consumption utility as one unit of the production good. Unlike the cash good, both deposits and cash can be used to pay for the production good. In what follows, we will use the terms “cash” and “money” interchangeably.

To introduce a motive for trade for all goods, we assume that no one can consume his or her own endowment or production, therefore everyone must trade to consume. All trades require payment one period ahead in cash or deposits. This means that in order to consume a cash good that is produced at date t, the buyer has to pay cash to the seller at date t-1. If he wants to consume a production good, he *also* has the option of writing a check to the seller at date t-1, which will clear against the funds he has on deposit at date t. The seller can use the cash or deposit he receives at date t to buy goods for consumption at date t+1. This payment in advance constraint also applies to sales of bonds (to be described) and restructured loans. If banks do not fail, this constraint will not bind, because bank claims are acceptable for payment for all transactions except cash goods. Finally, if a bank issues claims (in exchange for cash, bonds or loans) at date t, they can be used to initiate transactions at date t.

3.2. The Fiscal Demand.

The government endows investors before date 0 with M_0 of money and nominal bonds maturing at date 2 with face value B_2.[10] When the bonds mature, the government extinguishes them by repaying their face value in new money or issuing fresh bonds maturing at date 4.

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[10] Equivalently, we could assume the government uses these to pay for goods it purchases from investors (we would just have to carry an additional term representing the fraction of initial endowment of goods bought by the government). The goods it buys were returned as public services at date 0.

The government taxes sales of produced goods at the rate t (assume now that C is the after-tax quantity produced from a project, so total nominal taxes due on a project that matures at date t are $\frac{tC}{1-t}P_{tt}$ where P_{tt} is the cash price of a unit of consumption at date t). Because cash goods may lie outside the formal economy, we assume they are not taxed (nothing depends on this). Taxes are due at the time of production and are payable in money.

The odd-numbered dates, 1 and 3, are introduced just for the purposes of making the payment and settlement explicit. They could be thought of as close to dates 2 and 4 respectively. Two simplifying assumptions are reasonable. First, all actions that are to take place at date 4 can be committed to at date 3, and similarly, actions at date 2 can be committed to at date 1. In particular, this assumption allows late entrepreneurs to borrow deposits at date 3 against what they will have at date 4 after repaying the bank loan (= $(1-\gamma)C$). They can use the resulting deposits at date 3 to purchase goods for consumption at date 4. Similarly, the banker can also monetize his date-4 rents. Second, bank capital can be used as a means of payment whenever deposits can (or investors can borrow deposits pledging the value of the bank capital they hold at dates 1 and 3). If it were not for these assumptions, we would need to introduce another date to clear purchases initiated at date 4.

3.3. Money and Prices

We now have all the apparatus we need to determine prices. Once we determine prices in this economy, the channel from real activity to prices to the value of nominal deposits and back to real activity will become clear. We will see that there is a possibility of the banking system "automatically" stabilizing aggregate liquidity shocks through changes in the real value of nominal deposits. The system may also be destabilizing. Interestingly, both the possibility of destabilization and the policy remedies to correct it depend on the type of money demand that dominates at that juncture.

Let P_{ij} denote the price in date i cash of a unit of date j consumption. For example, a transaction for date 4 goods initiated at date 3 in cash at price P_{34} yields the seller P_{34} units of cash at date 4. Let i_{jk} be the gross nominal interest rate between dates j and k.

Since both bank claims and cash can be used to pay for produced goods, effectively all assets held by the bank including bonds are available as means of payment for produced goods whenever banks are solvent. By contrast, only cash can pay for cash goods and for taxes. There is a competitive market for deposits, bonds and goods at each relevant date from date 0, subject to the payment in advance constraints. Since cash goods and produced goods offer equivalent consumption on the dates consumption is desired, their relative prices will impose no-arbitrage constraints on the banking system. This will be important in what follows.

Assume that the quantities of money and bonds do not change after date 2, and there are M_2 units of money and B_4 units of bonds leaving that date. Bonds mature into B_4 units of cash at date 4. At date 4, cash is useful only to pay taxes, so it will be accepted in payment for produced goods because the seller wants to use them to pay taxes. Define X_4 as the quantity of goods sold for date 4 delivery. The nominal sales (all sales, including those paid with bank claims) is $P_{34}X_4$, nominal tax owed is $tP_{34}X_4$, and the total supply of cash at date 4 is $M_2 + B_4$.[11] As a result,

$$P_{34} = \frac{M_2 + B_4}{tX_4}. \tag{2.5}$$

This is a very simple version of the fiscal theory of the price level (see Calomiris [1988], Cochrane (2001) and Woodford (1995))

[11] Implicit in this is that the price of produced goods in assignable date-4 deposits (what one could term P_{44}) is the same as its price in date-3 cash. Equivalently, the gross nominal interest rate on bonds and deposits between dates 3 and 4, i_{34}, equals 1. Suppose not and the rate banks paid on deposits were higher than 1. Then everyone would deposit their cash in banks and use deposits to buy goods. But for any bank to hold cash, the rate on deposits should be 1, else the bank would use any excess cash to pay down deposits. Similarly, for banks to hold cash and bonds, the rates of return on them should be equal. So the nominal rate, i_{34}, is 1 and cash, deposits, and bonds pay the same rate, as they will on all dates that cash has no special value. This also explains why i_{12} equals 1.

At date 2, the purchase of q_3 of cash goods can be initiated with the outstanding date-2 cash, M_2. Since agents who get utility from consumption after date 2 are indifferent between consumption at date 3 or date 4, a holder of date-2 cash will spend it at date 2 or 3 depending on where he can purchase greater consumption. So the real value of the money stock at date 2 will be the larger of its purchasing power in buying cash goods for delivery at date 3 or the value of holding it to purchase produced goods at date 3 for delivery at date 4.[12] The purchasing power of the money stock is: $Max\{q_3, \frac{M_2}{M_2+B_4}tX_4\}$ where the last term is quantity of goods the current money stock, M_2, can purchase for delivery at date 4. As a result, if P_{24} is the price of date 4 consumption in date-2 cash, the date-2 value of the money stock is :

$$\frac{M_2}{P_{24}} = Max\{q_3, \frac{M_2}{M_2+B_4}tX_4\}\text{, or equivalently: } P_{24} = \frac{M_2}{Max\{q_3, \frac{M_2}{M_2+B_4}tX_4\}}$$

Comparing the two terms within the curly brackets of the expression for $\frac{M_2}{P_{24}}$, we see that when $q_3 > \frac{M_2}{M_2+B_4}tX_4$ money is valued more for its role in paying for transactions at date 2 (it has a liquidity premium) than as a store of value – the transactions demand dominates. Since a depositor can withdraw cash on date 2 to make payments, for someone to leave their money in the bank (or hold a non-monetary asset), deposits must offer a gross nominal interest rate of $i_{23} = \frac{q_3}{\frac{M_2}{M_2+B_4}tX_4} > 1$, and this is also the nominal rate on bonds (because banks can trade

[12] Another way to see this is that so long as the price of cash goods for transactions initiated at date 2 is below the price of produced goods at date 3, money will be fully used up in buying cash goods. But once there is enough money such that the price of cash goods equals the price of produced goods, any money left over after buying cash goods will be used as a store of value till it can be used for purchasing produced goods at date 3. Therefore, money will effectively be valued in terms of its date-3 purchasing power. This is the intuition behind the max function.

bonds with each other in competitive market). Note that once we allow money to have transactions value, we depart from the pure fiscal theory of the price level.

To determine the price level at earlier dates to date 2, we have to determine the real value of all government liabilities at date 2. Since we already know the value of the money stock, we now determine the value of outstanding bonds.

Let the date-2 value in cash of bonds maturing to pay B_4 at date 4 be b_{24}. Then because of the competitive market for bonds, the date 2 cash value of these bonds

$$b_{24} = \frac{B_4}{i_{24}} = \frac{B_4}{i_{23} * 1} = \frac{B_4}{Max\{1, \dfrac{q_3}{\dfrac{M_2}{M_2 + B_4} tX_4}\}} = B_4 Min\{1, \frac{tX_4}{q_3}(\frac{M_2}{M_2 + B_4})\}.$$

The real value (in date-4 consumption) of government liabilities leaving date 2 then is[13]:

$$\frac{M_2 + b_{24}}{P_{24}} = \frac{M_2 + B_4 Min\{1, \dfrac{tX_4}{q_3}(\dfrac{M_2}{M_2 + B_4})\}}{P_{24}}$$
$$= Max\{q_3 + \frac{B_4}{M_2 + B_4} tX_4, tX_4\}.$$

Now let us use this to determine prices (or equivalently, find the real value of money) *before* date 2. Let the quantity of money and bonds outstanding between date 0 to 2 be constant at M_0 and B_2 respectively (we will later allow monetary policy to alter these quantities). At date 2, the existing money stock and new money repaid on maturing date-2 bonds can be used to pay date-2 taxes as well as "buy" M_2 and B_4. Because the initial investors only value consumption on or before date 2, the real interest rate (that sets the relative price of consumption on or before date 2 and consumption after), r_{24}, can be greater than one even when date 2 consumption is positive.

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[13] We substitute for P_{24} and use $Min\{1, \frac{tX_4}{q_3}(\frac{M_2}{M_2 + B_4})\} = \frac{tX_4}{q_3}(\frac{M_2}{M_2 + B_4})$ and $Max\{q_3, \frac{M_2}{M_2 + B_4} tX_4\} = q_3$ when the nominal interest rate exceeds 1 to simplify the expressions.

The value of maturing bonds and money in units of date 2 consumption goods purchased at date 1 is

$$\frac{M_0 + B_2}{P_{12}} = tX_2 + \frac{1}{r_{24}} Max\{q_3 + \frac{B_4}{M_2 + B_4} tX_4, tX_4\} \qquad (2.6)$$

where we use the real interest rate to transform units of date-4 consumption into units of date-2 consumption. At date 0, cash of M_0 can be used to purchase cash goods q_1 at date 1. So its real value in terms of date 2 consumption is[14]

$$\frac{M_0}{P_{02}} = Max\{q_1, \frac{M_0}{P_{12}}\}. \qquad (2.7)$$

Again, if $q_1 > \frac{M_0}{P_{12}}$, money is valued for its transaction services and the nominal interest rate paid by deposits and bonds from date 0 to 1 is

$$i_{01} = \frac{q_1}{\frac{M_0}{P_{12}}} = \frac{q_1}{\frac{M_0}{M_0 + B_2}[tX_2 + \frac{1}{r_{24}} Max\{q_3 + \frac{B_4}{M_2 + B_4} tX_4, tX_4]} > 1.$$

In all of these expressions, the price level and the nominal interest rate depend on both the state of nature (through transaction demands,q_1 and q_3, fiscal demands X_1 and X_2, and the effect of liquidity conditions r_{24}), and monetary policy variables (M_0, M_2, B_2, B_4). For fixed fiscal and monetary policies, the state continent price level implies a given state dependent real value of nominal bank liabilities. There are some simple conditions when such automatic contingency stabilizes the banking system against liquidity shocks. To see how this might work, we have to revisit our "real" banking system to introduce money and goods prices.

3.4. Revisiting the "real" model.

First, assume the bank continues to issue real deposits – where the holder of a deposit with face value d_0 is paid a sum of $d_0 * P_{t2}$ in cash if the deposit is withdrawn at any time t on or before date

[14] Since no one cares about consuming before date 2, in the absence of bank failures creating an artificial desire for goods, the real interest rate between date 0 and date 2 will be 1.

2 (where $P_{22} = P_{12}$, by the argument in footnote 11). The main difference now is that banks will also hold money and bonds initially and depositors will withdraw some cash to buy cash goods at date 0. Also, cash goods will add to the supply of goods that are available to satisfy the initial investors' demand for consumption. However, it is easy to transform this seemingly more complicated problem into the simple form we have already seen.

Let us assume without loss of generality that the ex ante identical banks hold all the money and bonds initially (and offer initial investors claims in return for keeping these in the bank). The cash withdrawn to buy cash goods at date 0 is $q_1 P_{02}$. The deposits left in the bank have claim to $d_0 - q_1$ goods at date 2. Then the value banker of type i can raise in date-2 goods is

$$\frac{M_0 - q_1 P_{02} + B_2}{P_{12}} + \left[\alpha^i \gamma C + \mu^i (1-\alpha^i) c + (1-\mu^i)(1-\alpha^i) \frac{\gamma C}{(1+k) r_{24}} \right] \tag{2.8}$$

The numerator in the first term is the cash value of financial assets the bank holds, and it has to be divided by the date-1 price to get the value of those assets in terms of date-2 consumption. The term in square brackets is the value of the bank's project loans. Adding back the consumption value obtained by date-0 withdrawers, q_1, and simplifying, we get the value available to pay bank claimants to be[15]

$$v^i(P_{02}, P_{12}, \mu^i, r_{24}) = \frac{M_0}{P_{02}} + \frac{B_2}{P_{12}} + \left[\alpha^i \gamma C + \mu^i (1-\alpha^i) c + (1-\mu^i)(1-\alpha^i) \frac{\gamma C}{(1+k) r_{24}} \right] \tag{2.9}$$

Suppressing prices in $v^i(P_{02}, P_{12}, \mu^i, r_{24})$, we get analogous expressions to (2.1), (2.2), and (2.4).

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[15] Add q_1 to both sides and focus on the term $\frac{M_0 - q_1 P_{02}}{P_{12}} + q_1$. If $i_{01}>1$, $M_0 - q_1 P_{02}$ =0, and the term equals $\frac{M_0}{P_{02}}$. If i_{01}=1, $P_{02} = P_{12}$, so the term is again $\frac{M_0}{P_{02}}$.

The B type banker's problem is then

$$\max_{\mu^B} \quad \frac{M_0}{P_{02}} + \frac{B_2}{P_{12}} + \left[\alpha^B \gamma C + \mu^B (1-\alpha^B) c_1 + (1-\mu^B)(1-\alpha^B) \frac{\gamma C}{r_{24}} \right] \tag{2.10}$$

s.t.
$$v^B(\mu^B, r_{24}) \geq \frac{\max_{\mu'} v^B(\mu', r_{24}) + d_0}{2} \tag{2.11}$$

The equilibrium market clearing condition for date-2 consumption (real liquidity) is

$$\theta^G \frac{\max_{\mu'} v^G(\mu', r_{24}) + d_0}{2} + (1-\theta^G) \frac{\max_{\mu'} v^B(\mu', r_{24}) + d_0}{2} \leq q_1 + \frac{1}{1-t} \left[\theta^G C + (1-\theta^G) \left(\alpha^B C + (1-\alpha^B) \mu^B c \right) \right] \tag{2.12}$$

In sum then, if banks survive, the equilibrium prices, credit, and interest rates are obtained from solving (2.10) s.t. (2.6), (2.7), (2.11), and (2.12). In an appendix available from the authors, we present the full maximization problem as well as the clearing conditions with the cash in advance constraints. It is easily checked that the prices derived in the previous sub-section, the nominal interest rates, and the real interest rate r_{24} do indeed clear the market. Lemma 1 and Proposition 1 continue to hold, as we show in the appendix.

In summary then, augmenting the basic model with prices, financial assets, and cash goods does not change the basic insight that in the face of aggregate liquidity shortages, banks curtail credit and may even fail, severely dampening production. The point this makes clear is that credit contraction and failure are essentially real phenomena and occur when the bank is squeezed between non-renegotiable demand deposits fixed in real terms and a limited production of consumption goods. This then suggests that a potential way to avoid liquidity squeezes is to allow demand deposits to be denominated in cash, so that their real value could fluctuate with the price level. If the price level rises when there are production delays, nominal deposits could offer a hedge against aggregate shortages. Under special circumstances, this is indeed the case.

3.5. Nominal Deposits as a Hedge against Aggregate Liquidity Shortages.

Let banks issue nominal deposits of face value δ_0 outstanding at date 0 instead of real deposits. These give the depositor the right to withdraw δ_0 units of cash on demand. Deposits will return the nominal rate, i_{01}, if rolled over till date 1.

3.5.1. Fiscal demand dominates.

First consider a situation where the fiscal demand dominates. When there is plenty of money relative to cash goods, at the margin money is valued only for its role in paying taxes (the fiscal demand prevails). It is easy to see from (2.6) that

$$P_{02} = P_{12} = \frac{M_0 + B_2}{t\left[X_2 + \dfrac{X_4}{r_{24}}\right]} \tag{2.13}$$

So prices are inversely proportional to the present value of taxes, which is a constant function of discounted real production. Now nominal deposits serve as a hedge: the real payment they entail adjusts via the price level to the available amount that a "representative bank" can pay. If the real amount that a bank can collect on loans is fixed (or is relatively invariant with prices), and if bank asset portfolios are reasonably similar, price level changes will eliminate liquidity shortages, and the resulting credit squeezes and bank failures.

To see this, consider a "representative" bank (with $\alpha^i = \overline{\alpha} = \theta^G * 1 + (1-\theta^G) * \alpha^B$). Net of what they can buy with their financial assets, banks have to find additional real goods at date 2 of $\dfrac{\delta_0 - (M_0 + B_2)}{P_{02}} = \dfrac{\delta_0 - (M_0 + B_2)}{M_0 + B_2}(tX_2 + \dfrac{tX_4}{r_{24}})$ to pay off their depositors, where X_2 and X_4 are the total (taxable) output of produced goods at dates 2 and 4 respectively. P_{02} is from (2.13), when the availability of cash goods is small relative to other real quantities so that money does not have a transactions demand at the margin, and is priced only for its value in paying taxes.

The banking system's ability to pay depositors on date 2 is increasing in the fraction of projects that are early. If all projects are early, then $\alpha^B=1$, $X_2=\frac{C}{(1-t)}$, $X_4=0$, and the bank collects γC on its loans. For the bank to be able to repay when all projects are early, we require that $\gamma C \geq \frac{\delta_0-\left(M_0+B_2\right)}{M_0+B_2}\frac{tC}{(1-t)}$, or simply that

$$\frac{\delta_0-\left(M_0+B_2\right)}{M_0+B_2}\frac{t}{(1-t)}<\gamma<1 \tag{2.14}$$

Interestingly, once there is some α^B at which the representative bank survives, we can show the representative bank will never fail, no matter what the aggregate liquidity shock, that is, no matter what the aggregate $\bar{\alpha}$. To see this, note that for the bank to survive, we require

$$\bar{\alpha}\gamma C+(1-\bar{\alpha})[\bar{\mu}c+(1-\bar{\mu})\frac{\gamma C}{(1+k)r_{24}}]\geq\frac{\delta_0-(M_0+B_2)}{P_{02}}=\frac{\delta_0-(M_0+B_2)}{M_0+B_2}(tX_2+\frac{tX_4}{r_{24}})$$

where the left hand side of the inequality is the date-2 value of the representative bank's real assets based on the aggregate amount of restructuring, $\bar{\mu}$. Expanding the right hand side,

$$\bar{\alpha}\gamma C+(1-\bar{\alpha})[\bar{\mu}c+(1-\bar{\mu})\frac{\gamma C}{(1+k)r_{24}}]\geq\frac{\delta_0-(M_0+B_2)}{M_0+B_2}\frac{t}{1-t}\{\bar{\alpha}C+(1-\bar{\alpha})[\bar{\mu}c+(1-\bar{\mu})\frac{C}{r_{24}}]\}$$

Given (2.14), this is certainly true for $\bar{\mu}=1$.Since there is at least one feasible level of restructuring that leaves the bank solvent, the representative bank will not fail because it will select a $\bar{\mu}\in[0,1]$ such that it is solvent.

Proposition 2: If there is no transactions demand for money and there is some $\bar{\alpha}$ such that the representative bank can pay off its depositors, then the representative bank that makes real loans will not fail when it issues nominal deposits no matter what the actual realization of $\bar{\alpha}$.

Intuitively, when loans are real and deposits are nominal, when the value of money is determined primarily by the present value of real activity (as in the fiscal theory), and when there are no significant differences between bank portfolios (so that each one of them is a microcosm of the overall productive economy), banks are well hedged against aggregate liquidity shortages. An incipient shortage increases the price level and reduces the real value required to be paid out on the nominal deposits, thus alleviating the shortage. Nominal deposits are automatic stabilizers.

The requirements for this result are fairly stringent: First, we require each bank's loan portfolio to be representative of aggregate economic activity, that is, bank portfolios are identical. If not, while the aggregate banking sector is hedged, individual banks are not. If there is variation in α^i across bank portfolios, then banks with low α^i can fail, even when they have issued nominal deposits. Of course, given our results that prices adjust to aggregate liquidity, there exists a set of cross-subsidies from high α banks to low α banks that will keep all the banks alive. These cross-subsidies may not be privately rational for a healthy bank but may be in the collective interest.

Second, repayments on loans to projects should vary in proportion to aggregate real output. This would be true if loan contracts specify repayments in real terms or if borrowers typically promised to pay a high nominal amount, which is invariably renegotiated down to a real amount based on the real threat of foreclosure (and the real value of underlying collateral assets). It would not be true if loans entailed modest nominal repayments.

Third, for the fiscal demand to provide a value of nominal claims that is proportional to the discounted value of real production, taxation should also be proportional to aggregate production (for example, a linear sales tax).

3.5.2. A Well-Behaved Dominant Transactions Demand

All this was derived in the context of a dominant fiscal demand for money. We can obtain a similar result to Proposition 2 if the transactions demand were "well" behaved. If the quantities of cash goods are positively linearly related to the quantities of produced goods (e.g., when the real economy flourishes, so does the illegal economy), so $q_1 = \phi X_2$ and $q_2 = \phi X_4$ for $\phi > 0$, the price level again adjusts to offset real liquidity shocks. We will then have

$$P_{24} = \frac{M_2}{Max\{\phi, \frac{tM_2}{M_2 + B_4}\}X_4}$$

and $\frac{M_0}{P_{02}} = Max\{\phi X_2, \frac{M_0}{M_0 + B_2}[tX_2 + \frac{1}{r_{24}} Max\{\phi X_4 + \frac{B_4}{M_2 + B_4} tX_4, tX_4\}\}]$. This is bilinear in X_1 and X_2. If the fiscal demand is small (t close to zero), then the date 0 price level is $P_{02} = \frac{M_0}{\phi Max\{X_2, \frac{X_4}{r_{24}}\}}$. Following the logic above, this is a case where a nominal deposit contract with a fixed money supply again provides a good automatic hedge against aggregate liquidity shocks.

For both the example of dominant fiscal demand or this well behaved dominant transactions demand, prices vary inversely with available real liquidity, banks that issue nominal deposits have built in state contingency into their deposit liabilities even though the state of liquidity cannot be directly contracted on. We have already noted a number of stringent conditions for this to be true.

3.6. Nominal Deposits and Destabilizing Transactions Demand.

To see an alternative where nominal deposits do not serve as a hedge when there is a dominant transactions demand, let the quantity of cash goods at date 1, q_1, be a constant instead of being a fraction of date-2 production. Because a dominant transactions demand means the value of money is set by its value in purchasing cash goods, and cash goods do not vary with the state, nominal deposits have a fixed real value for a given money stock.

At date 0, depositors can withdraw up to δ_0 in cash to buy the cash good, but they can also roll over their deposit at the prevailing nominal rate, i_{01}. Since banks set the nominal rate $i_{01} = \frac{P_{12}}{P_{02}}$ to make depositors indifferent between withdrawing cash to buy cash goods at date 0 and paying with deposits for delivery at date 2, it must be that the real value a bank has to pay out at date 2 is $\frac{\delta_0 * i_{01} * i_{12}}{P_{12}} = \frac{\delta_0 * \frac{P_{12}}{P_{02}} * 1}{P_{12}} = \frac{\delta_0}{P_{02}}$.

Instead of the real value of deposits being determined by the price of produced goods for purchase at date 1, the real value is determined by the price of cash goods at date 0. But this price may have no relationship to aggregate real liquidity conditions in the economy. Instead, it will depend on the quantity of money (financial liquidity) relative to available cash goods. For example, when M_0 is not too large or q_1 is high, $P_{02} = \frac{M_0}{q_1}$. Therefore, a bank is solvent if and only if $\max_{\mu^i} \quad v^i(P_{02}, P_{12}, \mu^i, r_{24}) \geq \frac{\delta_0 q_1}{M_0}$. The aggregate liquidity constraint is obtained by substituting $d_0 = \frac{\delta_0 q_1}{M_0}$ in (2.12).

Intuitively, if depositors are promised a fixed amount of cash rather than a fixed real value, their real claim at date 2 depends on the outside opportunity depositors have to spend cash between dates 0 and date 2. In our model, the only outside opportunity they have is to buy cash goods, so if the price of cash goods is low, the real burden of deposit claims on banks becomes very high. Moreover, the real burden of repayment is now a function of the ex ante contracted level of nominal deposits, the quantity of cash goods available for purchase at date 0, and the quantity of money held by the banks, M_0, none of which are necessarily sensitive to the realized fraction of early projects, $\bar{\alpha}$.

The banking system may now be worse off issuing nominal demand deposits. Worse still, if the quantity of cash goods, q_1, is not constant but has a negative correlation with $\bar{\alpha}$ (for example if the illegal economy expands when the legal economy is anticipated to be down), the real deposit burden on banks issuing nominal deposits will be high precisely when they have the least resources to pay. By contrast, the repayment burden if real deposits had been issued would be constant across states, and this will result in lower bank failures. An example may be useful in bringing all this together.

Example:

Let the fraction of banks of type G be θ^G=0.3 and those of type B be 0.7. Let $\alpha^B = 0.25$, $c = 0.8, C = 1.6, \gamma = 0.8, k = 0.15$. Let M_0=0.2, B_2=0.4, $q_1 = 0.3$. Plugging in values $\overline{R} = 1.60$ and R=1.39 . Let the level of outstanding real deposits per unit invested in the bank at date 0 be d_0=1.4.

In the absence of any restructuring, the total supply of goods for early consumption is just 1.19. But outstanding deposits are 1.4, so at least some late projects have to be restructured to meet the liquidity demand. It turns out that the real interest rate r_{24} has to rise to 1.53 and $\mu^B = 0.57$ for aggregate liquidity demand to equal aggregate liquidity supply.

In figure 2, we plot how aggregate credit and real interest rates change as aggregate liquidity goes up with and increase in the fraction of G banks, θ^G. As the figure indicates, credit (=$1-\mu^B$) increases while the real interest rate falls.

Now let the fraction of G banks, θ^G, be constant at 0.3 but let more of the B type banks' projects be early so that $\alpha^B = 0.3$. Interestingly, even though aggregate liquidity goes up and B type banks now offer more credit (i.e., continue more late projects) so that $(1-\mu^B) = 0.51$, the real interest rate is now higher at 1.58. The reason for the higher real rate is that the B type banks now have higher date-2 value, so they can bid a higher rate for deposits and thereby reduce the fraction they have to restructure. Therefore the greater available real liquidity gets partly "absorbed" in greater credit (which goes up from 0.43 to 0.51) and partly in greater consumption by the now-richer date-0 investors (also see Figure 3).

Of course, if aggregate liquidity is too low, the B type banks might have to fail before aggregate demand for liquidity comes into balance with aggregate supply. As α^B falls, the B type bank's value falls until at $\alpha^B = 0.13$ the bank is insolvent even at the lowest interest rate r_{24}=R=1.39 required to give banks the incentive to restructure. When a bank is anticipated to fail, depositors run on it at date 0, and all its projects are restructured, including early ones.

Finally, note that the real interest rate, r_{24}, does not increase monotonically with aggregate liquidity. The real rate can increase if greater aggregate liquidity comes from an increase in date-2 production by the B type banks who bid up the interest rate, or decrease if it comes from an increased proportion of G type banks. Even if real deposit contracts were to offer a payout that was not constant but instead monotonic in the economy-wide real interest rate, the contract that maximized the ex-ante amount pledged to initial investors would not necessarily lead to allocations without bank failures. For any such contingent deposit contract, there exist many distributions of states of nature where aggregate shortages would still exist.

Now consider nominal deposits. Let α^B=0.25, let the level of nominal deposits be $\delta_0 = 0.933$. For reasons of space, let us skip the case with no transactions demand, where the price level adjusts so that banks stay solvent no matter what α^B is. With $M_0 = 0.2$ and $q_1 = 0.3$, $P_{02} = 0.66$ and $\frac{\delta_0}{P_{02}} = 1.4$. Thus for the initial parameters, the real burden of deposit repayments is the same as in the previous example, 1.4, and the resulting credit extended, $(1-\mu^B)$ = 0.43, is the same. It is easily seen from the earlier example that the bank will now fail if $\alpha^B = 0.13$, even though it has issued nominal deposits – because the real deposit burden is determined by P_{02} and does not adjust sufficiently with $\bar{\alpha}$.

Alternatively, let the available cash goods, q_1, go up to 0.31. Since the price level P_{02} drops, the real deposit burden increases when the bank has issued nominal deposits, and credit falls to $(1-\mu^B)$ = 0.38. When q_1 goes further up to 0.32, the B type banks fail (cash goods are so cheap that banks cannot pay enough to keep depositors in -- reflecting a true curse of plenty). By contrast, when banks have issued real deposits, an increase in q_1 only increases the available goods for date-2 consumption without increasing the real deposit burden. As a result, available credit increases, first to 0.44 for q_1 = 0.31 and then to 0.45 for q_1 = 0.32. In sum then, banks that issue nominal deposits can be extremely vulnerable to changes in outside cash opportunities because cash opportunities make nominal demand deposits effectively real but in a way that their value need not depend on $\bar{\alpha}$.

3.7. Discussion.

The point we have made is worth elaborating. When a bank offers nominal deposit contracts, its real repayment burden depends on monetary conditions and the state contingent movement in the price level. Too little money in the system can make cash transactions extremely lucrative. Real interest rates on deposits have to rise because depositors have the right to withdraw cash. Not only does this make the bank highly susceptible to fleeting opportunities available in the cash market even if that market is quite small, it also forces a future real liability on the bank (and a potential aggregate real liquidity shortage), even if the bank has issued nominal deposits. The nature of the bank's liabilities is critical to why it is susceptible to monetary fluctuations.

Important prior work has noted unaccommodated shocks to money demand can cause bank panics when banks issue nominal liabilities (see Champ, Smith and Williamson [1996]). There are a number of differences in our paper. First, unlike in our paper, the underlying real conditions in Champ et al. [1996] are fixed and not endogenous to bank decisions.

Second, in their model a fraction of investors have a direct demand for cash (because they are moving elsewhere). By contrast, in our model investors have a derived demand for cash based on opportunities in the cash goods market. These differences matter. First, we can examine the effects of real and financial liquidity (money) on not just bank failures but also bank credit. Second, the correlation between opportunities in the cash market and (endogenous) aggregate real liquidity determines whether the banking system is stable or not. Third, even if only a small fraction of investors have a direct demand for cash, the combination of a real liquidity shortage and lucrative cash transactions can be lethal for banks though not necessarily for other financial firms (we elaborate on this shortly).

That real deposits can be more stable than nominal deposits may account for why some banking systems issue them. Recently, the banking system in Argentina had committed to repay depositors in dollars (effectively real deposits in a peso economy) but the country did not have the dollars, or could not commit to attract enough of them, to repay depositors. Faced with such a shortage, banks failed in 2002. The lesson some economists draw from this crisis is that the banking system should not have issued real (i.e., dollar denominated) deposits.

But issuing peso denominated deposits may not be a panacea unless the value of the peso in terms of both local goods and foreign exchange fully and always reflects the condition of the real economy and the banking system's ability to pay. But if the value of the peso fluctuated in ways that did not reflect the underlying state of the economy, then the real repayment burden on the banks issuing nominal deposits would have become much higher than would have been the case if it were fixed in real terms. For instance, if investors fear that the government will devalue the peso in the future or expect a speculative attack on it, then the nominal and real interest rate on peso denominated deposits would have had to rise to match the real opportunities available to depositors who could withdraw pesos to buy dollars and then foreign goods. The real repayment burden on the banks would then become a function of the maximum overvaluation of the cash, and the health of banks would become hostage to the country's competence and credibility in managing its exchange rate. If this were questionable to begin with, the country's banking system may have been better off issuing "real" deposits.

IV. The Channels of Transmission of Monetary Policy

4.1. Effects of Policy Changes

We have assumed that both monetary and fiscal policy remain fixed even in the face of shocks to aggregate production or to the supply of cash goods. We can examine the effects of policy changes that work through the banking system when there are nominal deposits.

First, when the fiscal demand dominates, the nominal interest rate is zero. So open market operations exchanging money for bonds will have no effect on the price level or aggregate credit. Only changes in the real tax rate or a "helicopter" drop of money will change the price level. By contrast, when the transactions demand dominates, money has a liquidity premium; the nominal interest rate is no longer zero, and exchanging money for bonds and vice versa will have real effects. Let us examine this in more detail.

4.2. The Channels of Transmission of Monetary Policy: The Liquidity Channel

In our model, a current shortage of money relative to cash goods causes an incipient drain on the banking system, which is averted only if the banks pay depositors a very high nominal rate. But this increases future bank obligations (aggregate real liquidity demand), which will be met by increases in the real interest rate, by curtailing credit and restructuring projects, and by bank failures.

This then suggests a "liquidity" channel through which monetary policy can affect real economic activity: ultimately it works through prices, interest rates, and bank balance sheets, all ingredients that have been separately identified by other theories of transmission (see Bernanke and Gertler (1995) or Kashyap and Stein (1994, 1997) for detailed surveys). But the reason is different from the traditional ones. Banks are special in transmission not because of reserve requirements, sticky prices, or special guarantees given to deposits, but because their demandable liabilities are non-renegotiable, and convert to cash on demand and because they have some control over production of their borrowers.

To see the channel working, consider an open market repurchase conducted by the monetary authority, which has the effect of increasing the date-0 money supply to $M_0 + \Delta$ and reducing the face value of outstanding date-2 bonds to $B_2 - i_{02}\Delta$ where Δ is a small number. Let the open market repurchase be announced after initial contracts are negotiated and projects initiated, and be executed so that banks have the added money at date 0. To focus on the pure effect of the open market operation, let no other exogenous parameter be changed at this or other dates.[16] Furthermore, let the parameters be such that at any date-2 price level, the total supply of consumption goods is not enough to meet the total demand without some restructuring by B type banks. If the system is in a unique equilibrium where both types of banks survive, we have

Proposition 4: So long as the gross nominal interest rate exceeds 1, the effect of an open market repurchase of bonds with money at date 0 is to increase the available credit (i.e., reduce the fraction of late projects restructured).

Proof: See appendix.

So long as $i_{02} > 1$, the effect of an increase in money will be to increase P_{02} ($=\frac{M_0 + \Delta}{q_1}$), reduce the nominal rate, and reduce the required real deposit payout at date 2, $\frac{\delta_0}{P_{02}}$. This will cause real aggregate liquidity demand at date 2 to fall. A reduction of aggregate real liquidity demand will tend to reduce the fraction of projects liquidated, or equivalently, enhance the supply of credit. The details of the proof show that changes in prices or interest rates do not offset the direct effect of the decrease in liquidity demand.

[16] This would mean that at date 2, we would have to inject (helicopter drop) a small amount of money, $\Delta(i_{02} - 1)$ to keep the quantity of money constant at $M_0 + B_2$.

Example continued

Consider again our base case example with $\alpha^B = 0.25$. An increase in the money supply at date 0 from 0.2 to 0.22 increases credit from $(1-\mu^B) = 0.43$ to $(1-\mu^B) = 0.53$.

Corollary 1: When the gross nominal interest rate i_{02} is 1 (the net nominal interest rate is zero), an open market repurchase of bonds with money at date 0 will have no effect on the amount of credit available at date 2.

When the nominal interest rate is 1, money and bonds are equivalent because the marginal unit of money provides no transaction services while bonds provide no interest – there is enough money at date 0 that the price of cash goods purchased at that date equals the price of produced goods at date 1. Any additional money issued by the monetary authority to repurchase bonds will be held by the banks as a store of value and not withdrawn by investors for date-0 transactions. Open market operations will have no effect on the date-0 price of cash goods, $P_{02,}$ and none on banks' real deposit burden at date 2.

Finally, a few notes. First, because a significant portion of bank liabilities is convertible on demand, banks are susceptible to temporary spikes in the transactions demand for money. By contrast, financial intermediaries with longer maturity liabilities are affected only if a substantial fraction of their liabilities mature together at a time of high transactions demand. A financial intermediary with longer-term liabilities that are diversified across maturities will be much less affected by temporary fluctuations in monetary conditions. Changes in monetary policy will have less of an effect on the activities of such intermediaries.

Second, we have modeled the temporary shock as one to money demand, coming from a surge in supply of cash goods. It could equally well come from a direct increase in the demand for cash (for instance, a flight to cash) or from temporary fluctuations in the supply of money.

Finally, note there are other channels through which an exchange of money for bonds can affect the real activity of banks. In particular, it could work by altering the real value of financial assets on bank balance sheets. We describe such a channel, which we call the financial asset channel, in the appendix. In practice, it may be a less important channel for banks.

4.3. Does our model square with modern facts?

The pattern of response of aggregate output to monetary expansion has been well studied in the United States (see, for example, Bernanke and Gertler (1995)). One fact that is at odds with our model is that prices do not adjust rapidly. While we think the fact that our model does not require sticky prices to obtain monetary transmission is a virtue, we need to ask how we could get transmission if prices were indeed sticky. It turns out that a simple extension is sufficient.

Consider a very simple search model where dealers first post prices at the beginning of the period and buyers search for a seller. There must be some rationing if the price cannot vary with supply or demand changes. Assume that the probability of a buyer meeting a dealer is an increasing function of the ratio of supply of cash goods to demand at the posted price, P_{02}. The nominal supply of goods at date 0 is q_1P_{02} and the nominal demand is the total amount of cash withdrawn at date 0 by depositors. An open market operation that increases the amount of cash that can be withdrawn by depositors will decrease the probability of each depositor finding a dealer. As this probability decreases, the return from withdrawing cash falls toward the return of holding cash to buy goods one period later, and the nominal interest rate on bank deposits between dates 0 and 1 will fall (as will the real rate because of sticky prices). Thus the real payout burden on deposits at date 2 falls with an expansionary open market operation, much as in the case of flexible prices.

4.4. Financial Contagion.

Our model offers a stark and transparent way of modeling the use of money in payments. This allows us to see the effects of alternative assumptions. For instance, suppose depositors who run on a bank will not accept deposits on other banks but will only take money (for instance, because they may need time to verify the quality of the bank they will deposit in). With this small change in assumptions, bank failures can become contagious through their effect on monetary conditions.

In particular, suppose now that type B banks are not expected to meet their nominal deposit obligations at date 2 and fail. Then they will be run immediately at date 0. They will pay out their cash reserves to depositors, but once the B type banks run out, they will have to sell assets. If the only asset that their depositors will accept is cash, the bank's assets must be sold for cash (and not for deposits in other banks). The sale of the bank's assets will lock up more cash in transactions, leaving less cash to buy cash goods. The cash good price will fall further to

$$P_{02} = \frac{M_0}{q_1 + \theta^B (c + B_2)} \tag{2.15}$$

where the denominator in (2.15) now also includes the value of restructured loans and bonds that the B type banks sell. This will make the purchase of cash goods even more lucrative, and force the G type banks to pay a yet higher rate to keep their depositors from moving to cash. [17] Given the higher effective payout to deposits at date 2, these type G banks could also fail if $v^G < \frac{\delta_0}{P_{02}}$.

The mechanism here resembles the contagious bank failures described by Friedman and Schwarz (1963): Depositors run on banks and take money, forcing banks to sell assets for cash, which renders the money supply inadequate for the quantum of real activity, forcing a further drop in the price level and still more bank failures.

Conclusion

We have introduced money in a real model of banking. We show that there is a connection between bank lending and monetary conditions, especially in situations where there is

[17] Note that the problem is not the price drop of the assets sold in the fire sale (also present in Diamond [1987] and Allen-Gale [1998]), but the increase in the interest rate required to keep money from being withdrawn for cash purchases.

a binding aggregate supply of real liquidity. We show that monetary intervention can have real effects even in a world without many of the traditional assumed frictions. We also show that while nominal deposits can, under fairly restrictive conditions, serve as a natural hedge, in general they do not insulate the bank against adverse shocks. In fact, a bank that has issued real deposits may be better hedged.

Turning to the scope for future work, we have made a number of strong assumptions to simplify our analysis. Initial investors cannot substitute at all for consumption between dates. The qualitative results will not change if they have a strong preference for date 2 consumption over date 4 consumption, except that they may be willing to postpone consumption once the interest rate rises enough. This will cap how destructive liquidity shortages can be.

The required repayment on deposits is never determined by the underlying value of bank assets (because bank deposits can never be renegotiated down). Therefore, bank deposits can be denominated effectively in nominal terms. By contrast, we assume that repayments on bank loans are determined by the underlying value of collateral, so they are effectively real even if denominated in nominal quantities. Of course, if the required nominal repayment on a bank loan is small relative to the underlying collateral, then the bank loan is effectively denominated in nominal terms. This is likely to be the case in cyclical upturns. So a fruitful extension would be to see how the value of a bank's loan portfolio responds to changes in monetary policy, and how this affects credit. One conjecture is that this will depend on the phase of the business cycle.

There are a variety of other simplifying assumptions: we have no limitation on transactions imposed by the availability of deposits in this model. We do not have long dated bonds. Fiscal policy is relatively static in our model. Finally, we do not have agents who are totally outside the banking system. Relaxing each of these is a realistic and potentially interesting extension that suggests scope for future work.

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Appendix

Proof of Proposition 1:

In equilibrium, three conditions have to be met at date 2: The B type bank should be just solvent, aggregate liquidity demand should equal aggregate liquidity supply (goods market clearing) and the money market should clear.

Let us write down the three conditions, expand them and collect terms.

For aggregate liquidity supply to be equal to aggregate liquidity demand, we must have

$$q_1 + \frac{1}{(1-\tau)}\left[\theta^G C + (1-\theta^G)\left(\alpha^B C + (1-\alpha^B)\mu^B c\right)\right] = \theta^G \frac{q_1 + \frac{B_2}{P_{12}} + \gamma C + d_2}{2} + (1-\theta^G)\frac{q_1 + \frac{B_2}{P_{12}} + (\alpha^B \gamma C + (1-\alpha^B)c) + d_2}{2}$$

(2.16)

Expanding and rearranging, we get

$$\underbrace{\frac{1}{(1-\tau)}\left[\theta^G C + (1-\theta^G)\alpha^B C\right] - \frac{1}{2}\left[\theta^G \gamma C + (1-\theta^G)\left[\alpha^B \gamma C + (1-\alpha^B)c\right]\right] + \frac{q_1 - d_2}{2}}_{k_1} + \frac{1}{(1-\tau)}(1-\theta^G)(1-\alpha^B)c\mu^B = \frac{B_2}{2P_{12}}$$

$$=> \qquad k_1 + k_2\mu^B = k_3\left(\frac{1}{P_{12}}\right) \qquad (2.17)$$

Next, we have the solvency condition for the B type bank, which requires that

$$q_1 + \frac{B_2}{P_{12}} + \left[\alpha^B \gamma C + \mu^B(1-\alpha^B)c + (1-\mu^B)(1-\alpha^B)\frac{\gamma C}{(1+k)r_{24}}\right] = \frac{q_1 + \frac{B_2}{P_{12}} + \left[\alpha^B \gamma C + (1-\alpha^B)c\right] + d_2}{2}$$

Rearranging, we have

$$\frac{B_2}{2P_{12}} + \underbrace{\left[\frac{q_1}{2} + \frac{\alpha^B \gamma C}{2} - \frac{(1-\alpha^B)c + d_2}{2}\right]}_{k_4} + (1-\alpha^B)c\mu^B + \frac{(1-\alpha^B)\gamma C}{(1+k)r_{24}} - (1-\alpha^B)\frac{\gamma C}{(1+k)}\left(\frac{1}{r_{24}}\right)\mu^B = 0$$

$$=> \quad k_3\left(\frac{1}{P_{12}}\right) + k_4 + k_5\mu^B + k_6\left[\frac{1}{r_{24}}\right] - k_6\left[\frac{1}{r_{24}}\right]\mu^B = 0 \quad (2.18)$$

Finally, for money market equilibrium, we have $\tau X_2 + \frac{q_3}{r_{24}} = \left(M_0 + B_2\right)\left(\frac{1}{P_{12}}\right)$.

$X_2 = \frac{1}{1-t}\left[\theta^G C + \left(1-\theta^G\right)\left(\alpha^B C + \mu^B(1-\alpha^B)c\right)\right]$. Substituting and regrouping, we have

$$\frac{t}{1-t}\left[\theta^G C + \left(1-\theta^G\right)\alpha^B C\right] + \frac{t}{1-t}\left(1-\theta^G\right)(1-\alpha^B)c\mu^B + \frac{q_3}{r_{24}} = \left(M_0 + B_2\right)\left(\frac{1}{P_{12}}\right)$$

$$k_7 + k_8\mu^B + q_3\left(\frac{1}{r_{24}}\right) = k_9\left(\frac{1}{P_{12}}\right) \qquad (2.19)$$

Equations (2.17), (2.18), and (2.19) are in 3 unknowns, $\mu^B, \frac{1}{r_{24}}, \frac{1}{P_{12}}$. Solving for $\frac{1}{r_{24}}$ and $\frac{1}{P_{12}}$ from (2.17) and (2.18) and substituting in (2.19), we get

$$(k_1 + k_4 + k_6 a) + \left(k_2 + k_5 + k_6 b - k_6 a\right)\mu^B - k_6 b(\mu^B)^2 = 0 \qquad (2.20)$$

where $a = \frac{1}{q_3}\left[\frac{k_9}{k_3}k_1 - k_7\right]$ and b $= \frac{1}{q_3}\left[\frac{k_9}{k_3}k_2 - k_8\right]$.

Proof of Proposition 1 (i):

Implicitly differentiating (2.20) w.r.t. θ^G, we get

$$\frac{d\mu^B}{d\theta^G} = \frac{-\left[\frac{dk_1}{d\theta^G} + \frac{dk_4}{d\theta^G} + \mu^B\left(\frac{dk_2}{d\theta^G} + \frac{dk_5}{d\theta^G}\right) + \left(1-\mu^B\right)k_6\left(\frac{da}{d\theta^G} + \mu^B\frac{db}{d\theta^G}\right)\right]}{(k_2 + k_5 + k_6 b - k_6 a) - 2k_6 b\mu^B} \qquad (2.21)$$

Some tedious but straightforward algebra shows that the numerator is negative, where we use the condition that some restructuring is needed at any price to prove that $k_1<0$.

To sign the denominator, note that it is in the form $2A\mu^B + B$ where the quadratic in (2.20) is $A\mu^{B^2} + B\mu^B + C = 0$. Some algebra shows that A<0, C<0 and B>0. Therefore there will be two positive roots for μ^B. If the equilibrium is unique, the greater root must exceed 1, and the only feasible root is the smaller root, $\mu^B = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$. This implies $2A\mu^B + B > 0$ so $\frac{d\mu^B}{d\theta^G} < 0$ or $\frac{d(1-\mu^B)}{d\theta^G} > 0$.

Proof of Proposition 1 (ii):

Implicitly differentiating (2.20) w.r.t. α^B, we get

(2.22)

$$\frac{d\mu^B}{d\alpha^B} = \frac{-\left[\overbrace{\frac{dk_1}{d\alpha^B} + \frac{dk_4}{d\alpha^B} + \mu^B\left(\frac{dk_2}{d\alpha^B} + \frac{dk_5}{d\alpha^B}\right) + \left(1-\mu^B\right)\frac{dk_6}{d\alpha^B}\left(a+\mu^B b\right)}^{E} + \left(1-\mu^B\right)k_6\left(\frac{da}{d\alpha^B} + \mu^B\frac{db}{d\alpha^B}\right)\right]}{(k_2+k_5+k_6b-k_6a)-2k_6b\mu^B}$$

Now $\left(a+\mu^B b\right) = \frac{1}{r_{24}}$ and $\frac{dk_6}{d\alpha^B} = \frac{-\gamma C}{1+k}$, therefore $\left(1-\mu^B\right)\frac{dk_6}{d\alpha^B}\left(a+\mu^B b\right) = -\left(1-\mu^B\right)\frac{\gamma C}{\left(1+k\right)r_{24}} > -\left(1-\mu^B\right)c$ whenever $r_{24}>R$ (which is the case since some restructuring is needed).

Expanding the terms under E and substituting from the above inequality, we get E is greater than

$$\frac{(1-\theta^G)C}{(1-\tau)} + \frac{1}{2}(1-\theta^G)c - \frac{1}{2}(1-\theta^G)\gamma C + \frac{\gamma C + c}{2} - \mu^B\frac{(1-\theta^G)c}{(1-\tau)} - \mu^B c - (1-\mu^B)c$$

Grouping terms, recognizing that $\gamma C > c$ and that $\mu^B < 1$, this is easily shown to be positive. It is straightforward to show that $\left(1-\mu^B\right)k_6\left(\frac{da}{d\alpha^B} + \mu^B\frac{db}{d\alpha^B}\right) > 0.$ So the numerator of (2.22) is negative and following the same logic as part (i), we get $\frac{d\mu^B}{d\alpha^B} < 0$ or $\frac{d(1-\mu^B)}{d\alpha^B} > 0$.

Proof of Proposition 1 (iii) (sketch):

As banks restructure more to cope with a low α^B (part (ii)), eventually $\mu^B = 1$ and now no more liquidity will be available through restructuring. Banks will have to fail if α^B falls further else the aggregate liquidity constraint will not be met.

<u>Proof of Proposition 4:</u>

When net nominal interest rates are positive, we have the real payout on deposits at date 2, $d_2 = \frac{\delta_0 q_1}{M_0}$. Clearly, this falls as M_0 increases. So we have to show that a fall in d_2 increases $(1-\mu^B)$, the amount of credit extended at date 2 by the B type banks.

Substituting and implicitly differentiating (2.20) w.r.t. d_2, we get

$$\frac{d\mu^B}{dd_2} = \frac{1 + \frac{k_6 k_9}{2q_3 k_3}\left(1-\mu^B\right)}{(k_2+k_5+k_6b-k_6a)-2k_6b\mu^B} \tag{2.23}$$

The numerator is easily shown to be positive. Since the denominator has been shown to be positive, $\frac{d\mu^B}{dd_2} > 0$, and therefore $\frac{d(1-\mu^B)}{dM_0}\Big|_{M_0+B_2=Const} > 0$.

The Financial Asset Channel: The Channels of Transmission of Monetary Policy

Recall we showed that the real value of the claims on the government held by the banks is

$$\frac{M_0+B_2}{P_{12}} = tX_2 + \frac{1}{r_{24}} Max\{q_3 + \frac{B_4}{M_2+B_4} tX_4, tX_4\} \tag{2.24}$$

where B_4 is the face value of date-4 maturing bonds issued at date 2 and M_2 is the quantity of money left after date-4 maturing bonds are issued ($=M_0+B_2-b_{24}$ where b_{24} is the present value of the date-4 maturing bonds issued at date 2). Keeping real activity constant, the expression indicates that as the quantity of bonds B_4 that are issued decreases from a level that nearly absorbs the entire money stock (so that M_2 is an infinitesimal amount) to 0, the total value of government claims held by the banks decreases from $tX_2 + \frac{q_3+tX_4}{r_{24}}$ to

$tX_2 + \frac{1}{r_{24}} Max\{q_3, tX_4\}$.

The decrease in the real value of government claims with expansionary open market operations is not because of seigniorage (government real revenues are assumed fixed): When there is only a minuscule amount of money outstanding, not only can current holders of money buy cash goods at a deeply discounted price, but also government bonds maturing at date 4 account for the lion's share of the public's claims on the government, so bonds capture the full value of real taxes. As the money stock increases, the dealers in the cash market at date 3 no longer have to sell goods for a deeply discounted price. Also bonds have to share the value of real taxes with holders of money. Therefore, as the amount of money outstanding increases relative to bonds, and as the nominal rate falls to 1, the real value of government liabilities (money plus bonds) holding real activity and real rates constant falls.[18] Of course, once the nominal rate falls to one, no further alteration in value is possible, and further open market operations lose all effect.[19]

Thus expansionary open market operations reduce the real value of government assets held by banks at date 2, and reduce aggregate liquidity demand by reducing the real value of the bank's liabilities. This leads to an expansion in bank credit for similar reasons to the ones discussed in the previous sub-section. The "financial asset" channel is probably weaker than the "liquidity" channel because the former works primarily by altering the value of bank liabilities such as capital that are most sensitive to bank asset values (unlike the latter which works by altering the real value of deposit payouts). In practice, non-deposit bank liabilities are less likely to be held for liquidity or transactions purposes, and thus the change in their value will have less of an effect on the aggregate demand for liquidity.

[18] Contrast this with the effect on nominal liabilities. Pushing down the nominal interest rate increases the nominal value of bonds and the total nominal value of government claims.

[19] Note also that the government's real revenues are unchanged if real output does not change (and government expenditure is fixed). So substituting interest-bearing liabilities for non-interest bearing ones does not result in a greater real claim on the government or in lower seigniorage profits. There are no sticky prices in our model. Open market operations simply transfer value from one set of agents to others but do not alter the aggregate real future payments by the government.

Figure 1: Real interest rates and aggregate credit with changes in theta

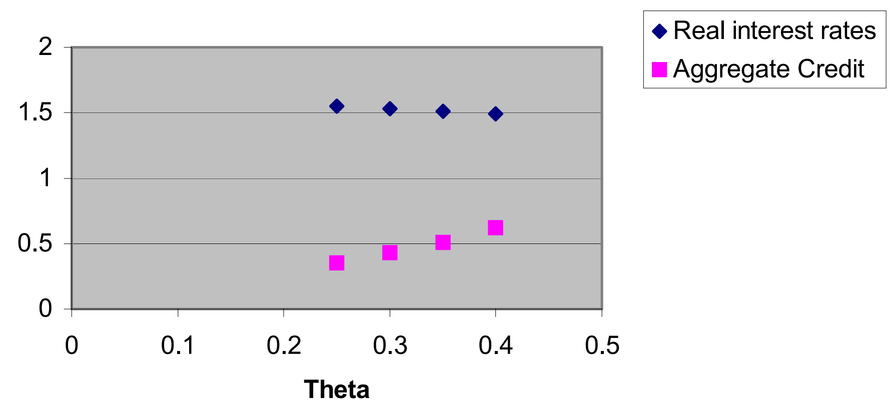

Figure 2: Changes in bank credit and interest rates with alpha

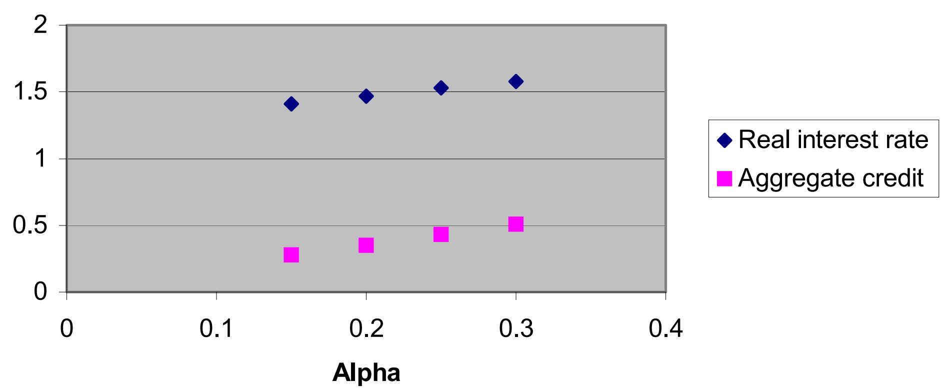

Figure 3: Time line of transactions:

Date -1
Banks offer interest rates on deposits and sells deposits and capital for initial investors' bonds and cash
Entrepreneurs receive loans (in bank claims)
Entrepreneurs buy goods from initial investors with bank claims

Date 0
State realized
Depositors withdraw cash to buy cash goods (if no more expensive than produced goods) or to hold as an asset.
If a bank faces withdrawals exceeding its cash, it sells bonds and restructured loans for date-1 delivery to meet withdrawal (similarly on all future dates)

Date 1
Cash goods sold at 0 delivered (similarly on all future dates)
Cash from date 0 good sales available to seller to deposit or spend (similarly on all future dates)
Cash from date 0 bank asset sales available for depositor to withdraw (similarly on all future dates)
Early entrepreneurs sell produced goods for deposits or cash

Date 2
Early entrepreneurs repay loans with deposits or cash
Early entrepreneurs pay taxes with cash from sales and withdrawn bank deposits
Banks pay dividend on capital (in deposits or currency)
Government repays maturing bonds in currency and issues new bonds
Cash is withdrawn to buy date-3 cash goods or to hold as an asset.

Date 3
Cash goods sold at date 2 delivered
Late entrepreneurs sell goods for bank claims and cash

Date 4
Late entrepreneurs repay bank with currency and deposits
Banks repay remaining net deposits in currency
Banks pay final dividend on capital in currency
Government repays maturing bonds in currency
All currency goes to pay taxes